# AD Model Builder Introductory Workshop
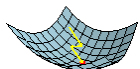
http://admb-project.org/

## Estimating and Expressing Uncertainty



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# Sources of uncertainty

- Process error — natural varibility in the world.

- Measurement error — elastic ruler, faulty electronics, observer bias, less than candid reporting of catch.

- Model specification error — it sometimes happens.

# ADMB tools for exploring uncertainty

- Inverse Hessian (aka Normal approximation).

- Profile likelihood

- Markov chain Monte Carlo (MCMC) sampling of the likelihood surface
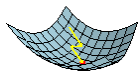
# Inverse Hessian

- Uses the "delta" method to compute the Hessian matrix, $H_{ij} = \frac{\partial^2 L}{\partial \Theta_i \partial \Theta_j}$

- $H^{-1}$ is an exact representation of the covariance matrix if the likelihood is quadratic near the minimum.

- Automatically computed for all model parameters, i. e. `init_` parameters decleared in the `PARAMETER_SECTION`.

- Optionally computed for any variable specified as `sdreport_` parameters in the `PARAMETE`

- Reported in the `*.std` and `*.cor` files.
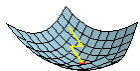
# sdreport variables

In the `.tpl` file:

```
DATA_SECTION
  init_number Nobs
  init_vector X(1,Nobs)
  init_vector Y(1,Nobs)
  init_number sd

PARAMETER_SECTION
  init_number a
  init_number b

  sdreport_vector Ypred(1,Nobs)

  objective_function_value f

PROCEDURE_SECTION
  Ypred=a+b*X;
  f=0.5*norm2((Y-Ypred)/sd);
```
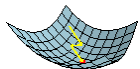
In the `.std` file:

```
index   name    value       std dev
    1   a       4.4667e+000 6.8313e+000
    2   b       9.9879e+000 1.1010e+000
    3   Ypred 1.4455e+001 5.8775e+000
    4   Ypred 2.4442e+001 4.9848e+000
    5   Ypred 3.4430e+001 4.1923e+000
    6   Ypred 4.4418e+001 3.5675e+000
    7   Ypred 5.4406e+001 3.2098e+000
    8   Ypred 6.4394e+001 3.2098e+000
    9   Ypred 7.4382e+001 3.5675e+000
   10   Ypred 8.4370e+001 4.1923e+000
   11   Ypred 9.4358e+001 4.9848e+000
   12   Ypred 1.0435e+002 5.8775e+000
```

# Parameter Correlations

In the `.cor` file:

```
The logarithm of the determinant of the hessian = -2.49496
index   name    value         std dev       1          2         3         4        5
    1    a      4.4667e+000 6.8313e+000    1.0000
    2    b      9.9879e+000 1.1010e+000   -0.8864   1.0000
    3    Ypred 1.4455e+001 5.8775e+000    0.9962 -0.8429   1.0000
    4    Ypred 2.4442e+001 4.9848e+000    0.9789 -0.7730   0.9929   1.0000
    5    Ypred 3.4430e+001 4.1923e+000    0.9311 -0.6565   0.9592   0.9860   1.0000
    6    Ypred 4.4418e+001 3.5675e+000    0.8207 -0.4629   0.8671   0.9202   0.9725  ...
    .
    .
    .
```
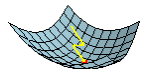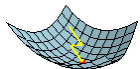
# Profile likelihood

- Delclare `likeprof_` variables for both model parameters and derived quantities.

- Specify likelihood profile computation at runtime with `-lprof` command line option.

- Results in the `*.plt` files.

# Controlling the profile likelihood calculations

```
PRELIMINARY_CALCS_SECTION
  a_prof.set_stepnumber(10);
  a_prof.set_stepsize(0.1);
```

Step is in standard deviation units

# Profile likelihood Report

In the `.plt` file:

```
Profile likelihood
  -18.467 0.000462415
  -17.4911 0.000800741
  -16.5152 0.00113907
  -15.5393 0.00147739
      .       .
      .       .
Minimum width confidence limits:
 significance level   lower bound   upper bound
       0.9                -8.59448      16.6654
      0.95                -10.6598      18.9582

One sided confidence limits for the profile likelihood:

Normal approximation
  -18.467 0.000461949
  -17.4911 0.000800021
      .       .
```
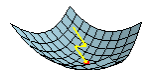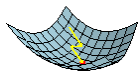
# Exercise: comparing estimates of uncertainty

- Modify the linear regression example to make a profile likelihood for the predicted Y on observation 3

- Compare the profile likelihood confidence intervals to those based on the normal approximation using the estimated standard deviation

# MCMC sampling of the likelihood surface