Likelihood concepts

ADMB and stock assessment

Arni Magnusson

ICES, 18 Oct 2012

Outline

1. Likelihood - relative probability, support, combine data sources
2. Estimation - MLE, log likelihood, confidence interval
3. Normal distribution - $N(\mu, \sigma)$, dnorm
4. Profile likelihood - procedure, interpretation

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Likelihood concepts

Relative probability

$$P(A_1) = 0.5 \qquad P(A_2) = 0.3 \qquad P(A_3) = 0.2$$

$$L(A_1) = 500 \qquad L(A_2) = 300 \qquad L(A_3) = 200$$

$$L(A_1) = 0.005 \quad L(A_2) = 0.003 \quad L(A_3) = 0.002$$

Likelihood concepts

Expresses how well the data **support** some parameter value or hypothesis

$$L\left(\theta \middle| \text{data}\right)$$

Like *RSS* but even more useful:

not only point estimate, but also **uncertainty**

Likelihood concepts

We can fit a model to many types of data at once and **combine** the likelihood components with simple multiplication

$$L = L_1 \times L_2 \times \cdots$$

Unified framework, for simple or complex models

Likelihood concepts

Choose between models with different number of parameters

$$2 \log \frac{L_1}{L_0} \sim \chi^2_{\Delta df}$$

$$\mathrm{AIC} = -2 \log L + 2k$$

$$\mathrm{BIC} = -2 \log L + \log(n) k$$

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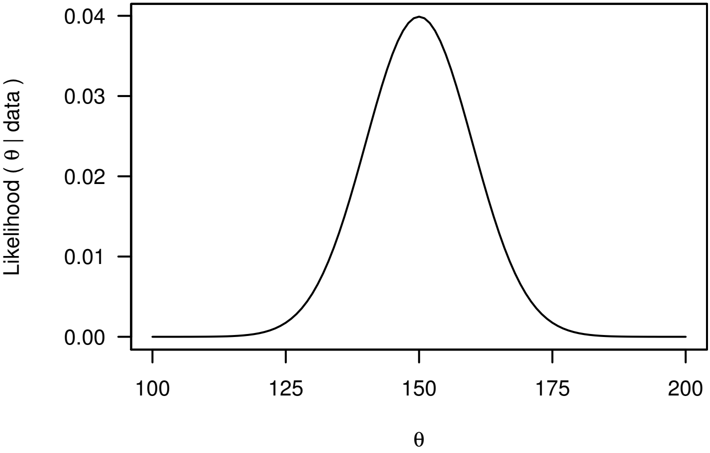
Maximum likelihood estimation



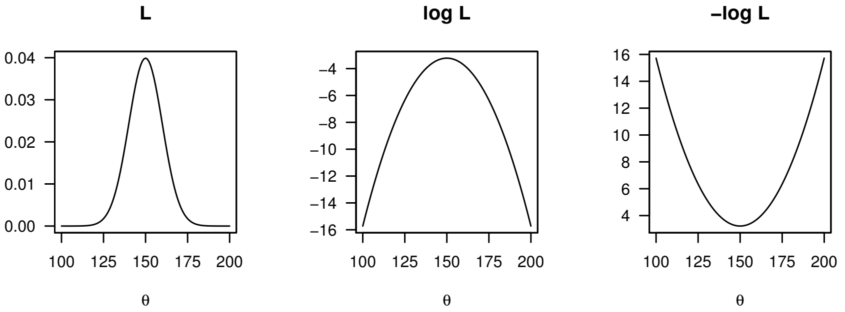
Log likelihood

Log transformation makes things easier

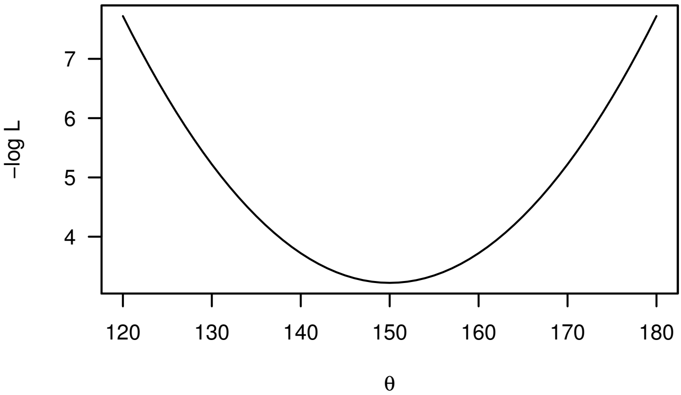
$$
\begin{aligned}
L(\theta|\text{data}) &= p(\text{data}|\theta) \\
& p(y_1, \ldots, y_n|\theta) \\
& p(y_1|\theta) \quad \times \quad \cdots \quad \times \quad p(y_n|\theta) \\
& \prod p(y_i|\theta)
\end{aligned}
$$

$$
\log L(\theta|\text{data}) = \sum \log p(y_i|\theta)
$$

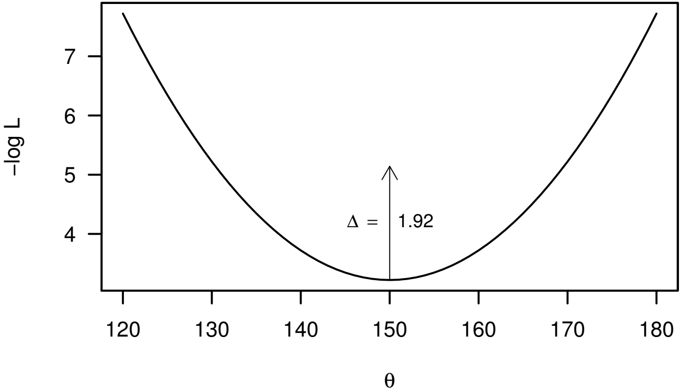
Log likelihood



Confidence interval

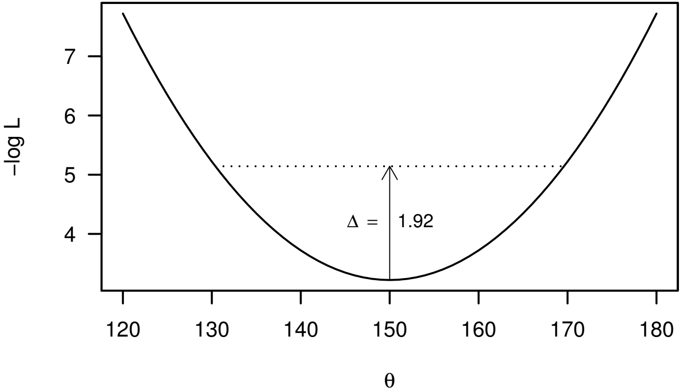


Confidence interval



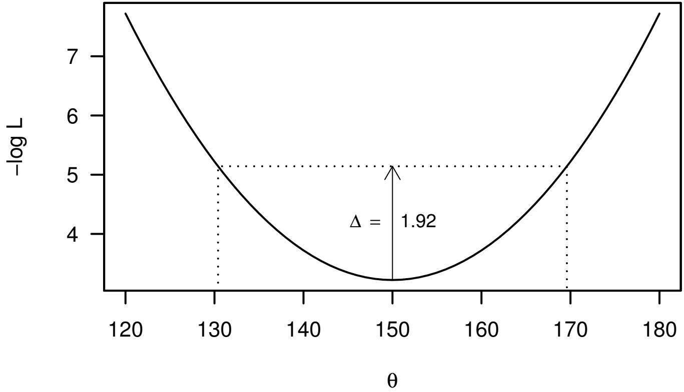
$0.5\chi^2_{df=1} = 1.92$ for 95% confidence interval

Confidence interval



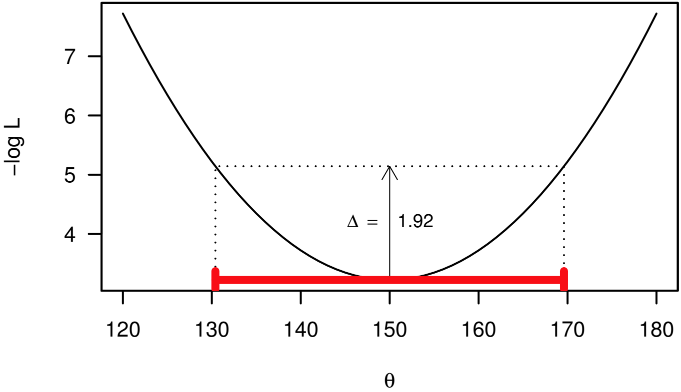
$0.5\chi^2_{df=1} = 1.92$ for 95% confidence interval

Confidence interval



$0.5\chi^2_{df=1} = 1.92$ for 95% confidence interval

Confidence interval



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Normal distribution

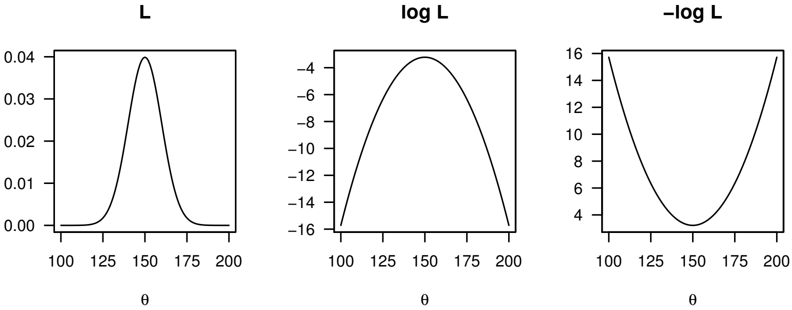
$$
\begin{aligned}
f(x) &= \frac{1}{\sigma\sqrt{2\pi}}\ e^{-\frac{(x-\mu)^2}{2\sigma^2}} \\
p(y_i|\theta) &= \frac{1}{\sigma\sqrt{2\pi}}\ e^{-\frac{(y_i-\mu_i)^2}{2\sigma^2}} \\
L(\theta|y) &= \prod\left(\frac{1}{\sigma\sqrt{2\pi}}\ e^{-\frac{(y_i-\mu_i)^2}{2\sigma^2}}\right) \\
-\log L &= \left[0.5n\log(2\pi)\right] \quad + \quad n\log\sigma \quad + \quad \frac{\sum(y_i-\mu_i)^2}{2\sigma^2} \\
&= \left[0.5n\log(2\pi)\right] \quad + \quad n\log\sigma \quad + \quad \frac{RSS}{2\sigma^2}
\end{aligned}
$$

dnorm in R

```
L <- prod(dnorm(y, mu, sigma))
```

```
neglogL <- -sum(dnorm(y, mu, sigma, log=TRUE))
```

dnorm in R




```
dnorm(theta,
m=150, s=10)
```

```
dnorm(theta,
m=150, s=10, log=T)
```

```
-dnorm(theta,
 m=150, s=10, log=T)
```

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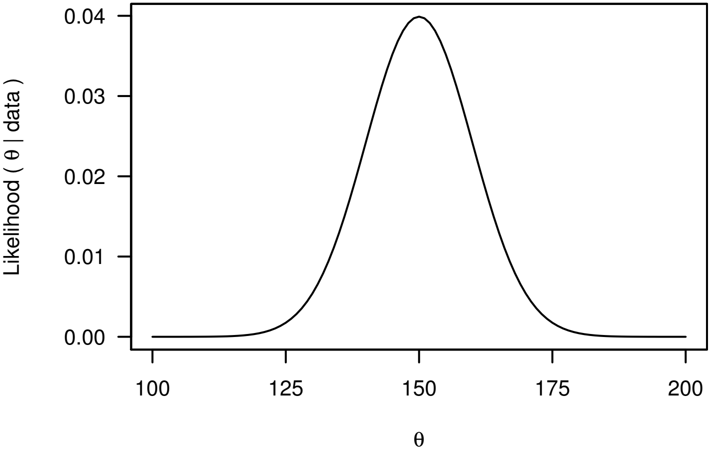
Profile likelihood

1. Fix θ (a parameter of interest) at some value
2. Minimize $-\log L$ by estimating all other parameters
3. Save this value of $-\log L$

Repeat over a range of θ values

Profile likelihood



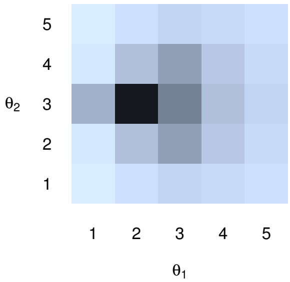
Interpretation

Consider a 2-dimensional likelihood surface, describing the likelihood at different values of two parameters:

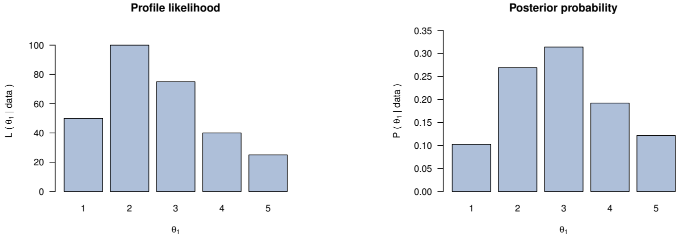
	$\theta_1 = 1$	$\theta_1 = 2$	$\theta_1 = 3$	$\theta_1 = 4$	$\theta_1 = 5$
$\theta_2 = 1$	5	15	25	20	15
$\theta_2 = 2$	10	40	60	35	20
$\theta_2 = 3$	50	100	75	40	25
$\theta_2 = 4$	10	40	60	35	20
$\theta_2 = 5$	5	15	25	20	15

Interpretation

Consider a 2-dimensional likelihood surface, describing the likelihood at different values of two parameters:



Marginal distribution



In likelihood inference, we find the maximum likelihood for each value of θ_1 across all values of θ_2

In Bayesian inference, we integrate the likelihood over θ_2

Interpretation

Would you place your bet on $\theta_1 = 2$ or $\theta_1 = 3$?

	$\theta_1 = 1$	$\theta_1 = 2$	$\theta_1 = 3$	$\theta_1 = 4$	$\theta_1 = 5$	sum
$\theta_2 = 1$	5	15	25	20	15	80
$\theta_2 = 2$	10	40	60	35	20	165
$\theta_2 = 3$	50	100	75	40	25	290
$\theta_2 = 4$	10	40	60	35	20	165
$\theta_2 = 5$	5	15	25	20	15	80
sum	80	210	245	150	95	780

profile, marginal, conditional, joint, . . .